A new model for the current–voltage output characteristics of photovoltaic modules

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Abstract

A computer program (PVCURVE) has been developed to simulate the d.c. output current–voltage (*I–V*) characteristics of a photovoltaic module, as a function of three characteristic data points which are usually provided by the manufacturer. These data points are: the open-circuit voltage (V_{oc}), the short-circuit current (I_{sc}), and the voltage and current at the maximum power point (V_m, I_M) for the module measured under standard test conditions of 1 kW/m^2 solar radiation and 25 °C cell temperature. The program can prepare plots of the *I–V* curves and determine the maximum power voltages and currents for the module under varying radiation levels and cell temperatures. If required, the *I–V* data points can also be given in a tabular form. The proposed model has been found to simulate the *I–V* characteristic of photovoltaic modules with a high degree of accuracy making it well suited for photovoltaic system design and performance analysis.

Introduction

Solar photovoltaic (PV) module and array designers need to be able to predict the effects of different types of shadowing and failures upon the performance of a PV module/array. These effects can then be minimized by varying the series/parallel grouping of cells/modules and the strategy for bypass diodes placement. One of the principal methods for assessing such failures, as well as PV system performance, is to plot the module/array current–voltage (*I–V*) characteristics under actual operating conditions. In a previous work [1], a computer simulation program was developed to predict the output *I–V* curves of commercial PV modules when operated under varying solar insolation levels and temperatures. The simulation relied totally on the manufacturer's electrical specifications and the *I–V* output curve of the module under standard test conditions (STC). It involved representing such curve with as many as 30 data points which have to be read from the given graph. In this paper, a new model which describes the *I–V* output characteristics of a solar cell/module/array is presented. The model is empirical and is put in a form which facilitates the analysis of PV systems without the need to use complex or tedious numerical solutions.

Theoretical background

Solar cells are basically p–n junctions which, when flooded by a stream of solar radiation, interact with the photons from the sun having energy greater than the band-

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gap energy of the semiconductor material. Photons generate electron–hole pairs either in the space charge region (junction) or in the semiconductor bulk where they may diffuse and reach the junction before recombining. Once at the junction, electron–hole pairs will be swept across by the junction field and become free to move through an external circuit and deliver power to a load. Modern cells are able to produce an open-circuit voltage of about 0.6 V and a short-circuit current density of about 0.38 mA/cm^2 [2].

Mathematical relationships can be found to help explain solar cell action. Since the p–n junction solar cell is essentially a diode, it can be anticipated that the solar cell equation will be a modified diode equation. The following equation is frequently used to describe the I–V characteristic of solar cells [3]:

$$I = I_L - I_0\{\exp[B(V + IR_s)] - 1\} \quad (1)$$

where I is the terminal current, V the terminal voltage, I_L the light generated current, I_0 the diode reverse saturation current, R_s the effective series resistance, and $B = q/nkT$ where q is the electronic charge, K the Boltzmann's constant, n the diode ideality factor, and T the absolute temperature.

Equation (1) is an implicit function and is non-linear in its parameters. It is known as the single exponential lumped-constant parameters model and has been accepted as being operationally sufficient to describe the I–V output characteristics of solar cells. The determination of the solar cell equation parameters is important, and must be done, first, if eqn. (1) is to be used in design calculations of PV systems. However, such determination is mathematically difficult and recourse to numerical solutions is inevitable. Moreover, a comparison between the experimental and the theoretical I–V curves, as described by eqn. (1), has shown some deviations to be present [4]. This is mainly due to the assumption of a constant series resistance value along the different portions of the I–V curve. Such assumption may be adequate if accuracy is not of prime importance and also if the translation of the I–V curve is over a relatively small intensity difference.

Several models for solar cells have been developed in an attempt to generate I–V output curves that closely approximate the measured ones [3–6]. One model [3] describes the I–V characteristic by the sum of two exponential terms with different saturation currents and diode ideality factors. The data points obtained in this manner showed a fair approximation to the experimental curve with some deviation at the knee of the curve. An improved lumped-constant parameters model together with the 'sum-of-two-exponential model' yielded much more satisfactory results but exhibits considerable complexity. Another approach [4] uses the single exponential model but treats the effective series resistance parameter as a variable that is a function of the terminal current, light intensity level, and cell temperature. Other models [5, 6] use curve-fitting techniques to determine the solar cell equation parameters from experimental data. However, these models may involve a large number of parameters that are difficult to determine [5]. Even when a smaller number of measured parameters is suggested [6], the solution, although much simpler, requires an iterative procedure and needs a reasonable starting guess which, if far from the optimal solution, may not converge and sometimes may even diverge.

In order to obviate the mathematical difficulties encountered in determining the solar cell equation parameters, a new model has been developed in this study. The model has a form which renders the analysis of PV systems much simpler without the need to employ the current practice of using numerical solutions.

Proposed model

Proposed in this study is a new and simple model which has been developed by observing the analogy and similarity between the I–V characteristic of a solar cell and the rise of current in an inductive circuit as depicted in Figs. 1(a) and (b). It can be seen from the analogy that the I–V output curve of a solar cell can be looked as an inverted inductor current function with the x-axis representing the terminal voltage instead of time. The inductor current starts rising at the open-circuit voltage point (V_{oc}) and reaches a final value equal to the short-circuit current (I_{sc}) when the terminal voltage V is zero. The I–V characteristic of a solar cell can then be described by:

$$I = I_{sc}\{1 - \exp[-(V_{oc} - V)/C]\} \qquad (2)$$

where C is a curve-fitting constant which takes the role of the time constant in an inductive circuit. It has the units of volts and its value is chosen such that eqn. (2) satisfies the maximum power point condition.

At the maximum power point $I = I_m$, $V = V_m$ and eqn. (2), when solved for C, yields:

$$C = (V_{oc} - V_m)/\ln[I_{sc}/(I_{sc} - I_m)] \qquad (3)$$

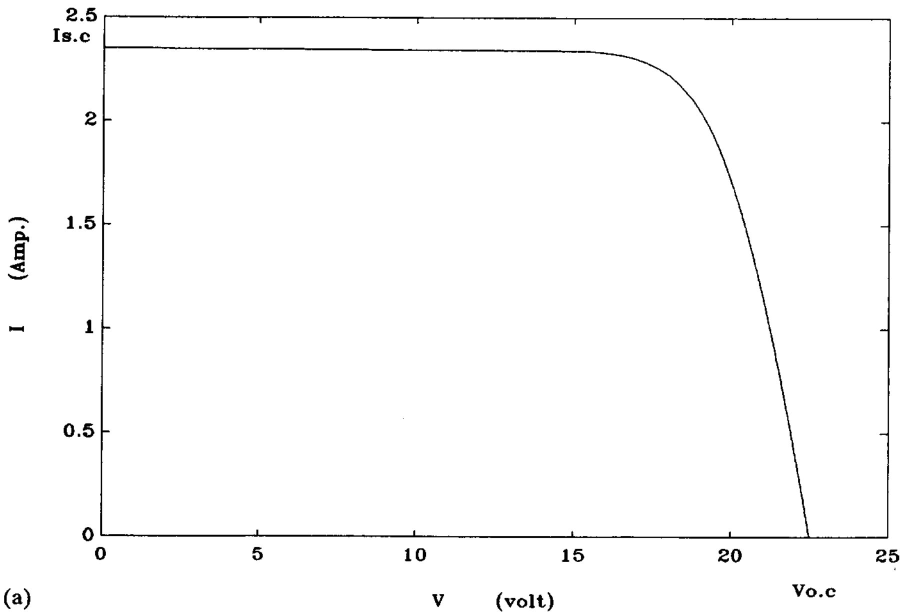
As seen from eqns. (2) and (3), the new model requires data defining three characteristic points of the cell: the open-circuit voltage (V_{oc}), the short-circuit current (I_{sc}), and the voltage and current at the maximum power point (V_m, I_m). The model fits an exponential curve to these three points and interpolates between them to provide a complete I–V curve. The above-mentioned basic parameters are usually provided by the manufacturer as I–V graphs corresponding to different radiation levels at standard temperature (25 °C), and to different temperatures at standard radiation level (1 kW/m^2). The constant C is determined by eqn. (3) using the manufacturer's electrical specifications at standard test conditions (STC). The value of C is then substituted in eqn. (2) to generate the I–V curve of the solar cell at STC.

It should be pointed out, however, that the proposed modelling approach is primarily a curve-fitting technique. The value of C does not represent any physical process but only provides the best average fit to the experimental I–V characteristic of the cell. A similar but slightly different model has been proposed earlier [7] and involved the evaluation of two curve-fitting constants instead of one.

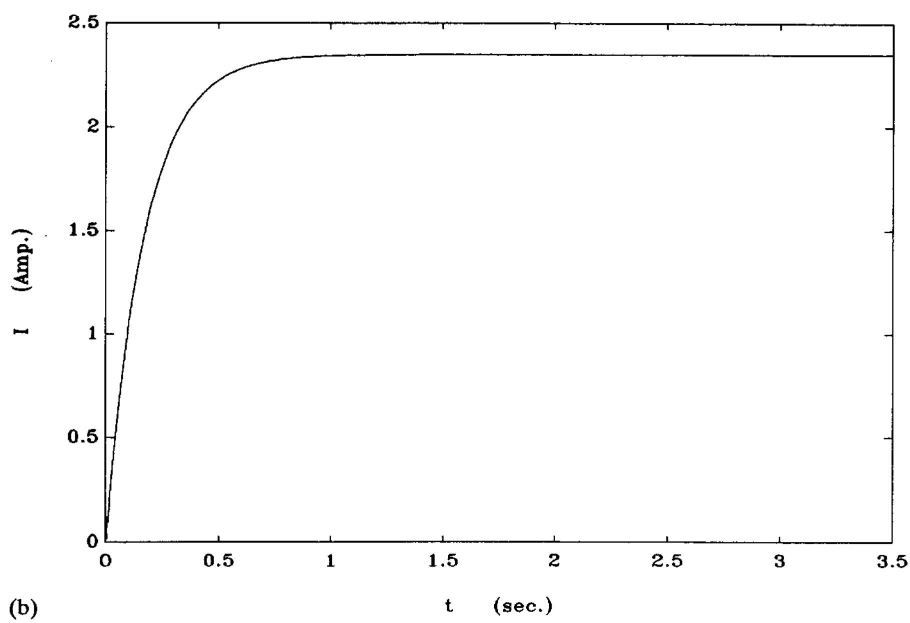
It should also be pointed out that the fit of eqn. (2) is suitable for a module and an array as well by simply scaling the I–V curves of each individual cell (i.e., multiplying the voltage by the number of cells in series and the current by the number of strings in parallel). This is because the I–V curves of a cell/module/array have similar shapes assuming all cells and modules to be identical.

Effects of radiation and temperature

The I–V curves of a cell/module/array vary with solar radiation L and temperature T. For flat plate PV modules without concentration, L varies from 0 to 1 kW/m^2 and T from 10 to 60 °C. In order to use the fit of eqn. (2), the values of V_{oc} and I_{sc} at any combination of L and T are needed. The value of the fitting constant C, on the other hand, is assumed to remain constant regardless of the changes that may occur in V_{oc}, I_{sc}, V_m and I_m due to radiation and/or temperature. A closer look at eqn. (3)



(a)



(b)

Fig. 1. The anology and similarity between the *I–V* curve of a (a) solar cell and (b) the rise of the current in an inductive circuit.

shows that this would be the case since the changes in both the numerator and the denominator does not alter the value of *C* significantly. The constant *C* is then calculated only once at the start of the simulation using the *I–V* basic parameter at STC.

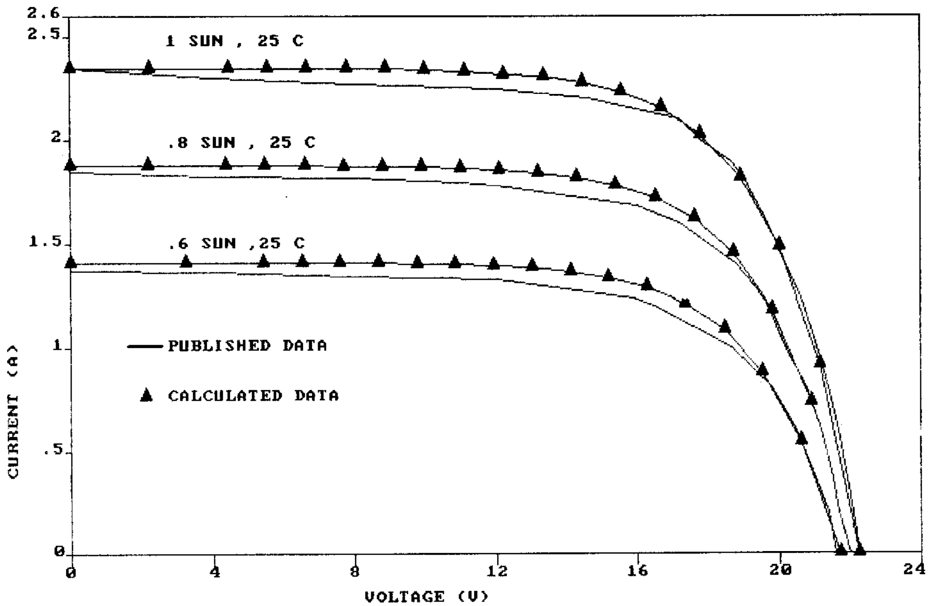


Fig. 2. Manufacturer's and calculated data for the SX-110 module under various intensity levels and 25 °C.

Since in most solar cells the short-circuit current I_{sc} is linearly proportional to the solar radiation level L [3], while the effect of temperature on I_{sc} is so small that it can be neglected [8], then the short-circuit current at any radiation level L and temperature T can be expressed as:

$$I_{sc}(L, T) = L I_{sc}(1, 25) \tag{4}$$

where $I_{sc}(1, 25)$ is the short-circuit current at 1 sun and 25 °C (i.e., at STC). The effect of radiation level on V_{oc} is logarithmic [9] and can be expressed mathematically by imposing the open-circuit condition on the single-exponential lumped constant parameters model of eqn. (1). The reason why we resort to eqn. (1), despite the limitations stated earlier, is that imposing the open-circuit conditions eliminates the effect R_s has on the equation which is the main reason causing the deviation between the experimental and the theoretical I–V curves as stated earlier. If this is done, the following equation results:

$$V_{oc}(L, 25) = (1/B) \ln[L I_{sc}(1, 25)/I_0] \tag{5}$$

The reverse saturation current I_0 is a property of the diode junction in dark and, hence, is independent on radiation level. It can be calculated only once by imposing the open-circuit condition on eqn. (1) at the STC:

$$I_0 = I_{sc}(1, 25) \exp[-B V_{oc}(1, 25)] \tag{6}$$

It is to be noted that eqns. (5) and (6) are applicable only for a single solar cell. For a module, scaling of each individual cell has to be applied, i.e., eqn. (5) has to be multiplied by the number of series cells/string in the module, and eqn. (6) has to be multiplied by the number of parallel strings in the module.

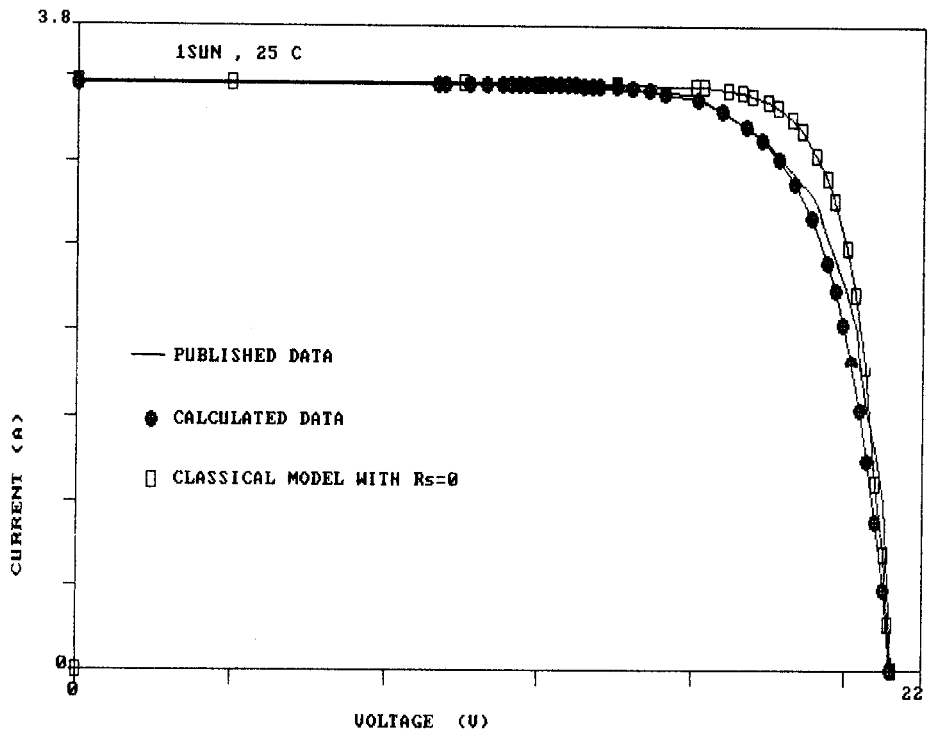


Fig. 3. Manufacturer's calculated data for the MSX-56 module at standard test conditions.

To take the effect of temperature into consideration, the open-circuit voltage as found from eqn. (5) has to be modified by a temperature coefficient α. A typical value of α for silicon solar cells is ± 2.2 mV/C above/below 25 °C [9]. Therefore, eqn. (5) is modified to be:

$$V_{oc}(L, T) = (1/B)\ \ln[LI_{sc}(1, 25)/I_0] - \alpha(T_c - 25) \tag{7}$$

where T_c is the cell temperature in °C. T_c is a function of the ambient temperature and the incident radiation level and can be estimated from the following equation [10]:

$$T_c = T_a + 30L \tag{8}$$

where T_a is the average daytime ambient temperature.

The procedure for generating the I–V curves of a PV cell/module/array under a given solar radiation level L and temperature T_a is summarized as follows. First, calculate a value for the curve-fitting constant C using manufacturer's given data at STC from eqn. (3). This value is calculated only once at the beginning of the simulation since it is assumed that it is radiation and temperature independent. Second, determine a value for I_0 using eqn. (6). This is straightforward if the manufacturer's given parameters are for a single cell. If, on the other hand, the given data are for a module, which is most likely to be the case, individual cell values are found by dividing the module open-circuit voltage by the number of series cells/string (N_s) and the module short-circuit current by the number of parallel strings (N_p). Once I_0 is found,

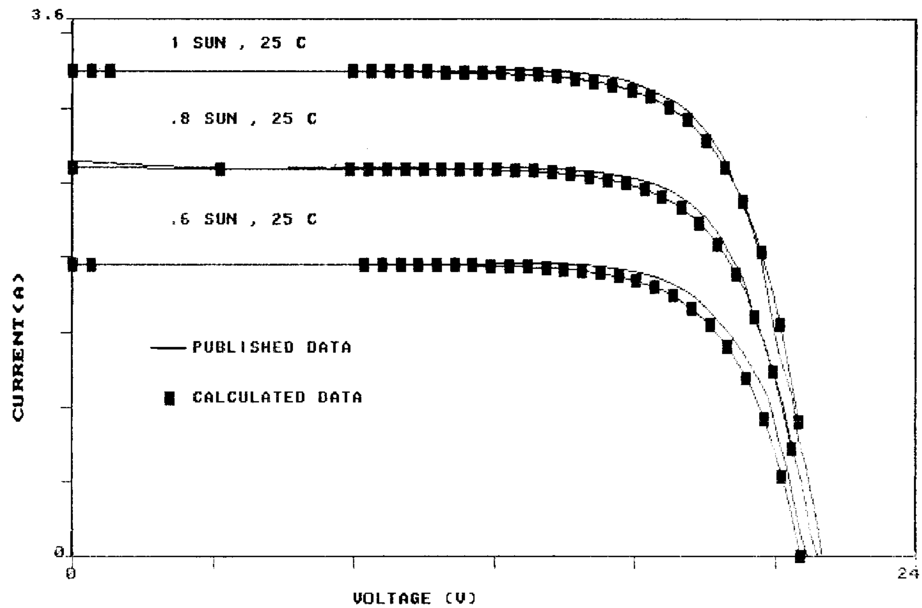


Fig. 4. Manufacturer's *I–V* curves compared with the fit found by PVCURVE for the LA361J51 module at various intensity levels and 25 °C.

eqn. (7) is used to calculate the open-circuit voltage/cell at any required radiation level and temperature. Similarly, eqn. (4) is used to calculate the short-circuit current/cell at the same conditions. Then, substitute the calculated values of V_{oc} and I_{sc} in eqn. (2) to generate the *I–V* curve or the solar cell at the required conditions. By scaling the *I–V* curve of the solar cell, *I–V* curves for a module/array can be generated.

Model implementation

The above-mentioned model has been implemented and programmed in BASIC. The program, called PVCURVE, requires the following parameters as input data: (i) module open-circuit voltage at STC, $V_{oc}(1, 25)$; (ii) module short-circuit current at STC, $I_{sc}(1, 25)$; (iii) module voltage and current at the maximum power point at STC, (V_m, I_m); (iv) number of cells/string N_s and number of parallel strings N_p; (v) required solar radiation level, and (vi) average daytime ambient temperature (this input parameter is presented as an option if the effect of cell temperature is to be taken into consideration). The program can prepare plots of the *I–V* curves or present them in tabular form; maximum power voltages and currents are also determined.

Model validation

In order to check the quality of the fit and to verify the validity of the model, the *I–V* characteristics for different types of PV modules were determined under varying radiation levels and temperatures and then compared with the corresponding data sheets supplied by the manufacturer. The following modules were tested:

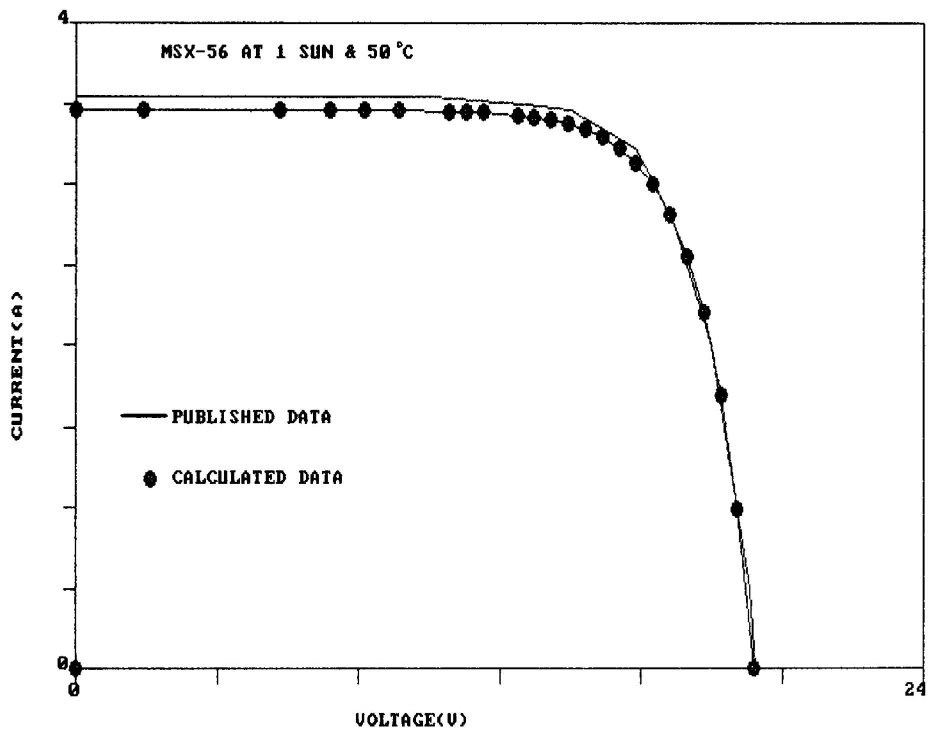


Fig. 5. Effect of cell temperature on published and calculated data for the MSX-56 module at 1 sun and 50 °C.

(i) the SX-110 (V_{oc}=22.25 V, I_{sc}=2.35 A, I_m=2.1 A, V_m=17.25 V, 40 cells in series) from Solarex;

(ii) the MSX-56 (V_{oc}=21.2 V, I_{sc}=3.46 A, I_m=3.16 A, V_m=17.7 V, 36 cells in series) from Solarex, and

(iii) the LA361J51 (V_{oc}=21.2 V, I_{sc}=3.25 A, I_m=3.02 A, V_m=16.9 V, 36 cells in series) from Kyocera.

Figures 2 to 6 reveal the good matching achieved between the proposed fit and the manufacturer's data sheets (which are laboratory curves) especially for the MSX-56 and the LA361J51 modules. These modules are characterized by an almost flat I–V behaviour at low voltage values. The temperature behaviour of the model is found satisfactory as demonstrated by Figs. 5 and 6 that show the good quality of the fit when temperature effects are considered, as compared with the manufacturer's laboratory curves. A small deviation is found near the short-circuit point due to the fact that the effect of temperature on I_{sc} is neglected, as stated earlier.

Discussion

As regard to accuracy, it is noted that the proposed model describes the I–V output characteristics for PV modules with sufficient accuracy that even exceeds that of the classical PV model. This was demonstrated when the model was tested on

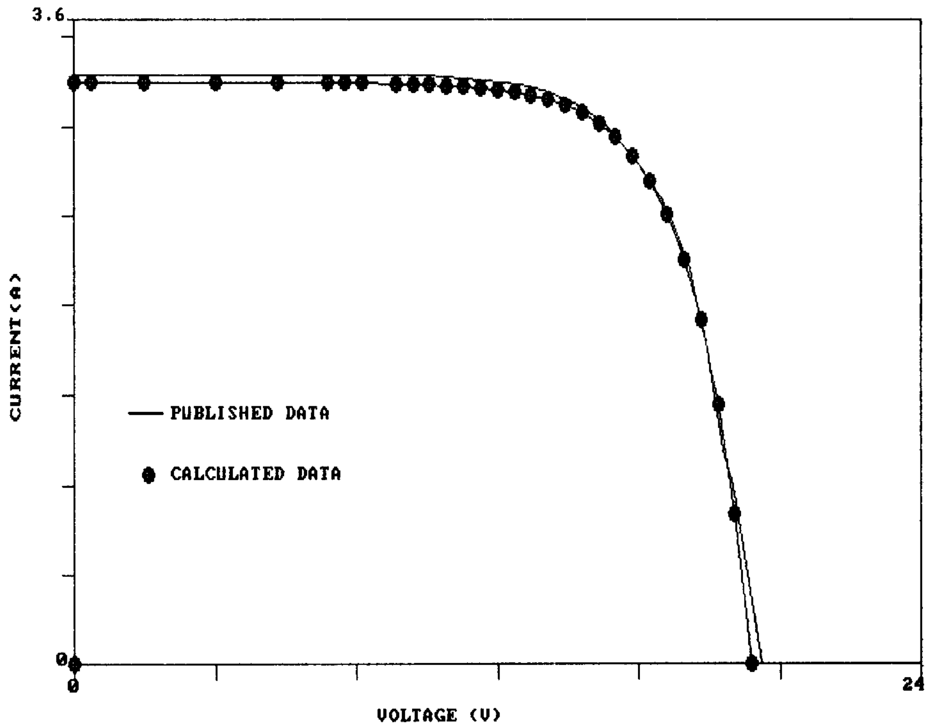


Fig. 6. Manufacturer's and calculated data for the LA361J51 module at 1 sun and 50 °C.

various types of PV modules where excellent matching between the calculated and the published data was observed. It should be noted, however, that the match is improved for those modules characterized by a flat *I–V* behavior at low voltage values. Nowadays, most newly developed modules fulfill this condition.

The proposed model has also the advantage of being explicit in form and thus the *I–V* output curve can be generated in a straightforward manner and in a much simpler way as opposed to the classical implicit form of eqn. (1).

Moreover, the new model is not a function of R_s. This, in turn, relieves the model from the necessity of calculating R_s first before attempting to generate the *I–V* curves. This is the most important advantage of the proposed model since not only it simplifies the procedure for generating the *I–V* curves but it also eliminates the dependence of the PV equation on R_s. Such dependence can greatly impede the characterization process of PV modules since the determination of R_s is not unique. Indeed, different values for R_s can be obtained through the application of the various methods used to determine the effective series resistance for the same module [4]. This would, in turn, lead to different *I–V* curves for the same module under the same conditions resulting in erroneous performance analysis.

It is to be noted that the proposed model, as given by eqn. (2), seems to be nothing but the classical PV equation given in eqn. (1) with R_s neglected. A closer look at the situation on hand shows that this is not the case simply because B is a constant parameter which has a numerical value different than $1/C$. Any attempt to use the classical PV equation with $R_s = 0$ would yield an output *I–V* curve that deviates even more from the laboratory curves. This is demonstrated in Fig. 3 by the curve-labelled classical PV model ($R_s = 0$).

Conclusions

The proposed model gives an effective and quite accurate tool for describing the output *I–V* characteristics of PV cells/modules/arrays under different radiation levels and temperatures. The model is based on three basic parameters only that are generally available by the manufacturer. It is simple and is written in a user-friendly manner that guides the user directly to the solution which may be presented in a tabular form or as a graph plot.

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